OCR
Oxford Cambridge and RSA

GCE

Mathematics B (MEI)

Unit **H640/03**: Pure Mathematics and Comprehension

Advanced GCE

Mark Scheme for June 2018

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

k Anything in the mark scheme which is in square brackets […] is not required for the mark to be earned on this occasio, but shows what a complete solution might look like.

Question			Answer	Marks	AOs	Guidance	
1			$[BC^2] = 32^2 + 14^2 - 2\times 32\times 14\cos 85°$	M1	3.1a	Use of cosine rule to find BC	1141.9…
			[BC] = 33.8 cm	A1	1.1		33.79…
			Perimeter = 79.8 cm	A1	1.1	Accept 80 provided 3 or more sf seen as BC	
				[3]			
2			$y = (x-1)^3 - 2(x-1) - 4$ oe	B1	1.1	Both factors $(x-1)$	$y = x^3 - 3x^2 + x - 3$ Must have $y =$ for 2 isw after correct answer
				B1	1.1	- 4	
				[2]			
3			AC = [AO – OC] $= 1 - \cos\theta$ or $\cos\theta = 1 - AC$	M1	1.1a	**AG** Allow AC = AO – OC with $OC = \cos\theta$ for M1	
			θ small so AC $1 - \left(1 - \frac{\theta^2}{2}\right) = \frac{\theta^2}{2}$	E1	2.1	Convincing completion	
				[2]			
4	(i)		$\frac{dy}{dx} = 1 - \frac{1}{(x-2)^2}$	M1	1.1a	Attempt to differentiate with one term correct	
				A1	1.1	Correct derivative	
			$1 - \frac{1}{(x-2)^2} = 0$ at stationary points	M1	1.1a		
			$x - 2 = \pm 1$ so $x = 1,\quad 3$	A1	2.2a	Both values of x	
			(1, –5) (3, –1)	A1	1.1	Both values of y - ft *their* x	
				[5]			

Question			Answer	Marks	AOs	Guidance	
4	(ii)		$\frac{d^2y}{dx^2} = \frac{2}{(x-2)^3}$ $x = 3$ $\frac{d^2y}{dx^2} > 0$ (2) so minimum $x = 1$ $\frac{d^2y}{dx^2} < 0$ (-2) so maximum	**M1** **A1** **A1** **[3]**	**1.1a** **2.4** **2.4**	OR Allow consideration of gradient either side of stationary point for **M1** Correct gradients above and below each tp **A1** Correct convincing conclusions (possibly with sketches) **A1** x f'(x) x f'(x) 0.5 0.56 2.5 -3.00 0.6 0.49 2.6 -1.78 0.7 0.41 2.7 -1.04 0.8 0.31 2.8 -0.56 0.9 0.17 2.9 -0.23 1 0.00 3 0.00 1.1 -0.23 3.1 0.17 1.2 -0.56 3.2 0.31 1.3 -1.04 3.3 0.41 1.4 -1.78 3.4 0.49 1.5 -3.00 3.5 0.56	
4	(iii)		$x = 2$	**B1** **[1]**	**1.2**		

Question			Answer	Marks	AOs	Guidance	
4	(iv)		$x > 2$	A1 [1]	2.2a		FT *their* (iii) if region is to right of *their* x value
5	(i)		$\log_{[10]} n = \log_{[10]} a + kt \log_{[10]} 2$ This is of form $y = mx + c$ [with $\log_{[10]} n$ as y and t as x]	M1 E1 [2]	1.1a 1.1	**AG** Allow $t \log 2^k$	
5	(ii)		Reasonable line of best fit drawn (by eye) Suitable method leading to a value eg use of intercept leading to $0.9 < \log a < 1.2$ So $7.4 < a < 15.85$ Suitable method leading to k value eg $k \log_{10} 2 =$ gradient ≈ 0.33 k in range $0 < k < 1.25$ and a in range $7.4 < a < 15.85$	B1 M1 M1 A1 [4]	1.1a 2.2a 1.1 2.2a	With $0.9 < c < 1.2$ Finding gradient of line or sub'n of t and log n	May use 2 points from line or condone use of 2 given points If gradient of exactly 1/3 used $k = 1.10730936...$
5	(iii)		$500000 = 10 \times 2^{1.1t}$ $1.1t \log 2 = \log 50000$ $t = 14.2$ $t = 14$ is 1/3/18 So 1/4/18	M1 A1 M1 A1 [4]	3.4 1.1 3.4 3.2a	Correct substitution Value of t (FT their a and k) Translation into date Rounding up	For k= 1.10730936.. t=14.1 Same answer

Question			Answer	Marks	AOs	Guidance	
5	(iv)		Suitable reason e.g. The data are only for a short time scale and cannot extrapolate e.g. There will not be enough people for the growth to continue	E1 [1]	3.5b		
6			${}_{15}C_5\left(x^2\right)^5\left(\frac{1}{x}\right)^{10}$	M1	3.1a	Identifying term with $\left(x^2\right)^5\left(\frac{1}{x}\right)^{10}$	Must see
			3003	A1 [2]	1.1		
7						**DR**	
			$5x-x^2=x(5-x)$	M1	3.1a	Factorisation	
			$[x=0],\quad x=5$	A1	1.1	Finding 5	
			The line does not go through the origin so $x=5$	E1	2.4	Rejection of origin as a point where they cross	May be later
			$y=4-kx$ so $0=4-5k$	M1	3.2a		
			$k=\frac{4}{5}$	A1	1.1		
			$4-\frac{4}{5}x=5x-x^2$	M1	1.1		
			$x^2-5\frac{4}{5}x+4=0$ OR $5x^2-29x+20=0$				
			$(5x-4)(x-5)=0$	M1	1.1		
			$\left(\frac{4}{5},\ \frac{84}{25}\right)$ o.e.	A1	2.2a	$\frac{84}{25}=3\frac{9}{25}=3.36$	
				[8]			

Question			Answer	Marks	AOs	Guidance	
8	(i)		$\frac{dx}{dt} = \frac{\left(1+t^3\right) - t\left(3t^2\right)}{\left(1+t^3\right)^2}$	M1	1.1a	**DR** Use of quotient rule – may be gained for $\frac{dy}{dt}$ if $\frac{dx}{dt}$ not seen (allow ±)	
			$\frac{dx}{dt} = \frac{\left(1-2t^3\right)}{\left(1+t^3\right)^2}$ oe	A1	1.1		
			$\frac{dy}{dt} = \frac{2t\left(1+t^3\right) - t^2\left(3t^2\right)}{\left(1+t^3\right)^2}$				
			$\frac{dy}{dt} = \frac{\left(2t-t^4\right)}{\left(1+t^3\right)^2}$ oe	A1	1.1		
			$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dt}{dx} = \frac{2t-t^4}{1-2t^3}$	M1	1.1a	Substitution into *their* $\frac{dy}{dx}$ dep on earlier M1	
			$t = 1 \Rightarrow \frac{dy}{dx} = -1$	A1	2.1		
				[5]			
8	(ii)		LHS = $\frac{t^3 + t^6}{\left(1+t^3\right)^3}$ oe	M1	1.1a	**AG** Expression for LHS	
			$= \frac{t^3\left(1+t^3\right)}{\left(1+t^3\right)^3}$	M1	1.1	Factorising	
			RHS = $\frac{t^3}{\left(1+t^3\right)^2}$ = LHS	A1	2.1	Completion of argument	
				[3]			

Question			Answer	Marks	AOs	Guidance	
9	(i)		$y = \dfrac{e^x}{1-e^x}$				
			$y(1-e^x) = e^x$	M1	1.1a	Clearing fractions	x and y may be interchanged at any stage
			$[\, y = e^x(1+y)\,] \quad e^x = \dfrac{y}{1+y}$	A1	1.1	Expression for e^x	
			$f^{-1}(x) = \ln\left(\dfrac{x}{1+x}\right)$	A1	2.1	Condone '$y =$' Condone no brackets or mod	
				[3]			
9	(ii)		$f^{-1}(x) \neq 0$	B1	1.2	Allow $y \neq 0$ but not $x \neq 0$	
				[1]			
10	(i)		$\overrightarrow{AC} = \begin{pmatrix} 2-a \\ 4-b \\ 2 \end{pmatrix}$, $\overrightarrow{AB} = \begin{pmatrix} 4-a \\ 2-b \\ 0 \end{pmatrix}$	M1	1.1	Forming vectors for sides AB and AC	Implied by next M1
			$(4-a)^2 + (2-b)^2 = (2-a)^2 + (4-b)^2 + 4$ o.e.	M1	1.1a	Use of AB = AC	
			$16 - 8a + a^2 + 4 - 4b + b^2 = 4 - 4a + a^2 + 16 - 8b + b^2 + 4$	M1	1.1	expanding	
			$4a - 4b + 4 = 0 \Rightarrow a - b + 1 = 0$	A1	2.1	**AG** Convincing completion	
				[4]			

Question			Answer	Marks	AOs	Guidance	
10	(ii)		D has position vector $\begin{pmatrix}3\\3\\1\end{pmatrix}$ where D is midpoint of BC	B1	3.1a	Midpoint OR if clearly minimising AC or AB - **M1** for relevant vector using a and b (May be implied by second M1)	
			$\overrightarrow{AD} = \begin{pmatrix}3-a\\2-a\\1\end{pmatrix}$	M1	1.1	Finding relevant vector in terms of a or b only	
			Area $= \frac{1}{2}AD.BC = \frac{2\sqrt{3}\sqrt{(3-a)^2+(2-a)^2+1}}{2}$	M1	1.1	Expression for AD or AD^2 (correct method but may have errors)	May use area proportional to AD, AC or AB without calculation of expression for area
			$\sqrt{3}\sqrt{2\left((a-2.5)^2+0.75\right)}$	M1	3.1a	Completion of square	Or differentiation of AD, AD^2, AC, AB, AC^2 or AB^2.
			$a = 2.5$ for min	A1	2.2a		
			Position vector $\begin{pmatrix}2.5\\3.5\\0\end{pmatrix}$	A1 [6]	3.2a		
11	(i)		The geometric mean cannot be calculated	E1 [1]	2.3		

Question			Answer	Marks	AOs	Guidance	
11	(ii)		The arithmetic mean will be less than the geometric mean	E1 [1]	2.3	E.g. The arithmetic mean will be negative	
12			$(a-b)^2 \geq 0 \Rightarrow a^2 - 2ab + b^2 \geq 0$	B1	1.1	Squaring bracket	
			$a^2 + b^2 \geq 2ab \Rightarrow a^2 + 2ab + b^2 \geq 4ab$ $(a+b)^2 \geq 4ab$	B1	3.1a	Adding $2ab$ to each side	
			$a+b \geq \sqrt{4ab} \Rightarrow a+b \geq 2\sqrt{ab} \Rightarrow \frac{a+b}{2} \geq \sqrt{ab}$	B1 [3]	2.1	Square root and correct completion	
13			Let the terms be $\frac{c}{r}, \quad c, \quad cr$	B1	3.1a	Expressions for three consecutive terms of a GP (any correct form)	
			The geometric mean of first and last is $\sqrt{\frac{c}{r}cr}$	M1	1.1	Expression for GM of first and last term (any correct form) *FT* their terms	
			$\sqrt{\frac{c}{r}cr} = \sqrt{c^2} = c$; this is the middle term	E1 [3]	2.1	**AG** Correct completion	
14	(i)		Angle BDC = $90 - \theta$ (angles of triangle)	M1	2.1	Including reason	Reasons can be given in either order
			Angle CDA = θ (Angle ADB = 90° as it is the angle in a semicircle)	E1 [2]	2.2a	Answer given so mark is for reason	

Question			Answer	Marks	AOs	Guidance	
	(ii)		Triangle ACD, $\tan\theta = \frac{h}{b}$ Triangle BCD, $\tan\theta = \frac{a}{h}$	M1	1.1	At least one correct expression for tan θ	Alternative method: triangle ACD is similar to triangle DBC
			$\frac{a}{h} = \frac{h}{b} \Rightarrow h^2 = ab \Rightarrow h = \sqrt{ab}$	E1 [2]	2.1	Setting expressions equal and correct completion to given answer **AG**	
15			Suppose that for the given perimeter, $4L$, there is a rectangle which is larger in area than the square.	M1	2.5	Setting up a statement for contradiction.	
			There is a square which has the same area as this rectangle but a smaller perimeter so its side is less than L.	A1	3.1a	Use of statement in line 31-32	
			The square with side L has perimeter $4L$ and an area larger than the given rectangle. This is a contradiction so the square must have the largest area of all rectangles with given perimeter.	A1 [3]	2.2a	Completion to correct conclusion, including contradiction	

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